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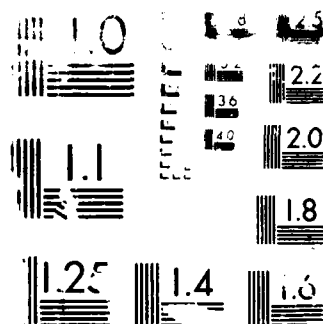
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ESTIMATING SYSTEM AND COMPONENT RELIABILITIES
UNDER PARTIAL INFORMATION ON CAUSE OF FAILURE

by

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and Thom J. Hodgson^{2,3}
University of South Carolina
Statistics Technical Report No. 136
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DEPARTMENT OF STATISTICS

The University of South Carolina
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ABSTRACT

Estimating component reliabilities along with the system reliability frequently requires using lifetimes from the system level. Due to cost and time constraints, however, the exact cause of system failure may be unknown. Instead, it may only be ascertained that the cause of failure is due to one component in a subset of components, e.g., the subset forms a subsystem. Confronted with such data, this article discusses how to exploit fully the available information using a maximum likelihood approach. We extend and clarify the useful work of Miyakawa (1984). A small Monte Carlo study indicates the helpfulness of this approach.

KEY WORDS: Reliability estimation; Partially masked cause of failure; Incomplete data; Maximum likelihood estimation; Reliability data bases.



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1. INTRODUCTION

Estimating component reliabilities along with the system reliability frequently requires using lifetimes from the system level. This can arise from field life data or from accelerated or nonaccelerated life testing data. Miyakawa (1984) remarked that, "... investigation of the cause of failure is expensive and requires time, and hence sometimes the cause of failure is not observed, even if the failure time is observed." Usher (1987) noted, "... when large computer systems fail in the field, analysis is usually performed such that a small subset of components, perhaps a circuit card, is identified as the cause of failure. In an attempt to repair the system as quickly as possible, the entire subset of components is replaced and the exact failing component may not be investigated further." See also Gross (1970). For related biological data compare Dinse (1986).

We use a parametric approach because in engineering problems realistic parametric models are often available. Also, parametric estimators of reliability can be much more efficient than nonparametric ones.

Data for which the cause of failure is narrowed to a subset of components (e.g., subset $\{1,2,3\}$), we call masked because the true cause of failure is partially masked from our knowledge. Note that this can be viewed as a type of censored data. Here the cause of failure is censored, but the system time may be complete (i.e., uncensored) or censored. We handle both cases related to time and masked data.

For the first case of masked but time complete data we derive its full likelihood in Section 2. This full likelihood extends and clarifies Miyakawa's

(1984) useful likelihood. From our full likelihood, we give a helpful partial likelihood and conditions for its proper statistical use. This provides further insight and understanding of Miyakawa's (1984) likelihood. In fact, if the conditions we give are not met then estimators based on Miyakawa's likelihood will actually be inconsistent. (I.e., the estimators will not converge in probability to the desired, true parameters.)

System lifetime censoring, of course, can occur with masked data. We, thus, develop the likelihood approach for that case in Section 3.

Although these techniques were developed in analyzing the reliability of such electronic products as monitors, graphic display terminals, modems, point of sale terminals, etc. (see the discussion in Usher (1987)), the actual data is unavailable for this article due to proprietary rights. In Section 4 a small Monte Carlo study investigates the effects of masking on the estimators. As expected, the mean square error and the bias get worse with more masking, and they improve with an increase in sample size.

Concluding remarks about analyzing masked data are presented in Section 5. We also comment on using these techniques in building better reliability data bases.

2. THE LIKELIHOOD WITH SYSTEM TIME COMPLETE DATA

We deal with the case of system time complete but masked data in this Section. We develop the full likelihood for this case when the system is a series of J components. (It should be noted that a similar development is possible if the system is parallel, "I-out-of- J ," or arbitrary.) The "components" could also be "modules" in series. See, for example, Barlow and

Proschan (1981) for more on modules.

Consider a sample of n systems. Let T_i be the random life of the i th system; $i=1, \dots, n$. Let T_{ij} be the random life of the j th component in the i th system; $j=1, \dots, J$. Note that

$$T_i = \min(T_{i1}, \dots, T_{iJ})$$

for $i=1, \dots, n$. We assume the T_{ij} 's are independent. For each fixed j the T_{1j}, \dots, T_{nj} represent a random sample from component j 's life distribution F_j . We assume F_j has a density (or mass function) f_j indexed by the parameter vector θ_j . For each j a different number of parameters in θ_j is allowed if needed. Let $\bar{F}_j(t) = 1 - F_j(t)$ be the reliability of component j at time t .

To precisely derive the likelihood involving masking, we need the following notation. Let K_i be the index of the component causing the failure of system i . (We assume the cause of failure K_i is unique, of course.) Note that K_i is a random variable. Also, K_i may or may not be observed. That is, the component causing the system to fail may be masked with other components in the system.

Before the sample, we are led to the minimum random subset, M_i , of components known to contain the true cause of failure of system i . In short, $K_i \in M_i$ and M_i is minimum. After the sample data is taken, we observe:

$$\begin{aligned} M_i &= S_i \subset \{1, 2, \dots, J\} \\ T_i &= t_i \end{aligned} \tag{2.1}$$

where $i=1, \dots, n$. If $S_i = \{j\}$, then we know $K_i = j$, and hence, the cause of failure is not masked. If, for example, $S_i = \{1, 2\}$, we have $K_i \in S_i$ but the

true value of K_i is masked.

From (2.1) the observed data can be expressed as $(t_1, S_1), \dots, (t_n, S_n)$. We now derive the full likelihood for this data. To make proper probability statements, we let each f_j be a probability mass function. This also helps develop a reader's intuition for masked data. The situation of f_j being a density is analogous, of course.

Consider (t_i, S_i) and its contribution c_i to the full likelihood L .

$$\begin{aligned} c_i &= P\{T_i = t_i, M_i = S_i\} \\ &= P \left[\bigcup_{j \in S_i} (T_i = t_i, K_i = j, M_i = S_i) \right] \\ &= \sum_{j \in S_i} P(T_i = t_i, K_i = j, M_i = S_i) \\ &= \sum_{j \in S_i} P(T_i = t_i, K_i = j) \cdot P(M_i = S_i | T_i = t_i, K_i = j) \end{aligned}$$

The expression $P(M_i = S_i | T_i = t_i, K_i = j)$ represents the conditional probability that the observed minimum random subset is S_i , given that system i failed at time t_i , and the true cause was component j . For $S_i = \{j\}$, the expression is the conditional probability the cause of failure is known. For S_i containing more than j , it yields the conditional probability of masking with the set S_i .

In our industrial problems, we found that masking usually occurred due to constraints of time and expense of failure analysis. Schedules often dictated that complete failure analysis to determine the true cause of system failure be curtailed. In this setting, we had for j' fixed and $j' \in S_i$

$$\begin{aligned} P(M_i = S_i | T_i = t_i, K_i = j') &= P(M_i = S_i | T_i = t_i, K_i = j) \quad (2.2) \\ \text{for all } j \in S_i. \end{aligned}$$

As a result, this term can be factored out of the summation to yield

$$\begin{aligned} c_i &= P(M_i = S_i | T_i = t_i, K_i = j') \cdot \sum_{j \in S_i} P(T_i = t_i, K_i = j) \\ &= P(M_i = S_i | T_i = t_i, K_i = j') \cdot \sum_{j \in S_i} [f_j(t_i) \prod_{\substack{s=1 \\ s \neq j}}^J \bar{F}_s(t_i)] \end{aligned}$$

The full likelihood under (2.2) is then

$$L = \prod_{i=1}^n c_i .$$

Note that these masking probabilities can be a function of time. Also, we allow $P(M_i = S_i | T_i = t_i, K_i = j') \neq P(M_i = S_i | T_i = t_i, K_i = j)$ for $j \notin S_i$. We assume only that the masking probabilities conditional on the time and cause are not functions of the life parameters. We state this for future reference as

$$P(M_i = S_i | T_i = t_i, K_i = j') \text{ does not depend on the life distribution parameters .} \quad (2.3)$$

This is analogous to a censoring distribution not depending on the life distribution parameters. Cf. Miller (1981).

Using (2.3), we write a reduced or partial likelihood

$$L_R = \prod_{i=1}^n \left\{ \sum_{j \in S_i} [f_j(t_i) \prod_{\substack{s=1 \\ s \neq j}}^J \bar{F}_s(t_i)] \right\} . \quad (2.4)$$

Under (2.2) and (2.3), maximizing L_R with respect to the life parameters is equivalent to using L . This is similar to the usual derivation of a time

censored (and not masked) data partial likelihood.

The above clarifies and extends Miyakawa's (1984) helpful likelihood. He had m systems with the cause of failure known (not masked), while $n-m$ were masked (with only the time of failure known). If m is random as in our industrial problems, then his likelihood is really a partial likelihood. (If for some reason m is a fixed number, our full and partial likelihood are the same).

We suggest it best to view his likelihood (as well as L_R here) as a partial likelihood that under the appropriate conditions will yield good, consistent estimators. Without (2.3), and all else true, for example, such a partial likelihood can yield inconsistent estimators. (I.e., the estimators will not converge in probability to the desired, true parameters.) For proper statistical applications, it is important to be clearly aware of the effects of masking probabilities and needed conditions.

3. THE LIKELIHOOD WITH SYSTEM TIME CENSORED DATA

Life testing, in general, can result in censored system life data. System time censoring occurred in the masked data we had. We present the likelihood for that case here. Let Y_i be the random censoring time associated with the i th system. Let $G_i(t) = P(Y_i \leq t)$, $\bar{G}_i(t) = 1 - G_i(t)$, and $g_i(t)$ be the density or probability mass function of Y_i . Note that we can handle each Y_i having a different censoring distribution. We also allow Y_i to be a fixed number if needed. We have

$$\delta_i = I(T_i \leq Y_i) = \begin{cases} 1 & \text{if } T_i \leq Y_i \text{ (uncensored)} \\ 0 & \text{if } T_i > Y_i \text{ (censored)} \end{cases}$$

and

$$X_i = \min(T_i, Y_i).$$

As is usual, we assume T_i and Y_i are independent and that $G_i(t)$ does not depend on the life parameters. Note that if $\delta_i=0$, then $M_i = \{1, \dots, J\}$ because we do not observe the cause of failure (as well as not observing the time).

The data is $(X_1, \delta_1, M_1), \dots, (X_n, \delta_n, M_n)$. By considering the two possibilities of $(X_i=t_i, \delta_i=1, M_i=S_i)$ or $(X_i=t_i, \delta_i=0, M_i = \{1, \dots, J\})$, we express under (2.2) the likelihood contribution of the i th observations as

$$c_i = \left[P(M_i = S_i | T_i = t_i, K_i = j') \cdot \sum_{j \in S_i} [f_j(t_i) \prod_{\substack{s=1 \\ s \neq j}}^J \bar{F}_s(t_j)] \right. \\ \left. \cdot (\bar{G}_i(t_i^-)) \right]^{\delta_i} \cdot \left[g_i(t_i) \prod_{s=1}^J \bar{F}_s(t_i) \right]^{1-\delta_i}, \quad (3.1)$$

where $\bar{G}_i(t_i^-) \stackrel{\text{def.}}{=} \lim_{t \rightarrow t_i^-} \bar{G}_i(t)$. Note that (3.1) holds for Y_i discrete or

continuous. (For Y_i continuous $\bar{G}_i(t_i^-)$ is simply $\bar{G}_i(t_i)$). The full likelihood

is simply

$$L = \prod_{i=1}^n c_i.$$

With the conditions stated above and (2.3), the reduced likelihood is

$$L_R = \prod_{i=1}^n \left\{ \left[\sum_{j \in S_i} [f_j(t_i) \prod_{\substack{s=1 \\ s \neq j}}^J \bar{F}_s(t_j)] \right]^{\delta_i} \cdot \left[\prod_{s=1}^J \bar{F}_s(t_i) \right]^{1-\delta_i} \right\}.$$

Again, maximizing the life parameters with L_R under the stated conditions is equivalent to using L .

The above likelihood is for randomly right censored data. For other types of censoring on the system lifetime (e.g., for interval censored), analogous likelihoods can be derived under appropriate conditions.

4. MONTE CARLO STUDY

We investigate via a small Monte Carlo study the effects of masking on the bias and mean square error (MSE) of the maximum likelihood estimators (MLE's) derived from L_R .

To cover simply both smaller and larger sample cases, we chose to simulate samples of size $n=10$ and 100 . Consider a series system of $J=3$ components (or modules). Components 1 and 2 form a subsystem. The reliability function is exponential, $\bar{F}_j(t) = e^{-\lambda_j t}$, for $t \geq 0$ and $j=1,2,3$. For easier comparisons, we used $\lambda_1 = \lambda_2 = \lambda_3 = 1$. The exponential random variables were generated using the inverse cumulative distribution function method; see Kennedy and Gentle (1980).

Simulating the effect of masking, we randomly masked the true cause of system failure based upon a total masking probability, (i.e., proportion). This total masking proportion was derived from the probabilities of $M_1 = \{1,2,3\}$ and $M_1 = \{1,2\}$. This meant we allowed partial masking where the cause of failure was known to be in the subsystem, $\{1,2\}$, or total masking occurred, $\{1,2,3\}$. No

other masking was allowed. To satisfy (2.2), we had $P(M_i = \{1,2,3\} | T_i = t_i, K_i = j)$ was constant for $j=1,2,3$ and $P(M_i = \{1,2\} | T_i = t_i, K_i = j)$ was constant for $j=1,2$. Condition (2.3) was also easily met by assigning the conditional probabilities without a functional dependence on the life parameters, $\lambda_1, \lambda_2, \lambda_3$.

Let n_1, n_2 and n_3 denote the number of failures where $M_i = \{1\}$, $M_i = \{2\}$, and $M_i = \{3\}$ respectively. Let n_{12} denote the count of $M_i = \{1,2\}$, the number of partially masked failures. Let n_{123} represent the number of totally masked, $M_i = \{1,2,3\}$, systems. The MLE's are found by using (2.4) to be

$$\hat{\lambda}_1 = \left[n_1 + n_{12} \left(\frac{n_1}{n_1 + n_2} \right) + n_{123} \left(\frac{n_1^k}{n_1^k + n_2^k + n_3^k} \right) \right] / \sum_{i=1}^n t_i$$

$$\hat{\lambda}_2 = \left[n_2 + n_{12} \left(\frac{n_2}{n_1 + n_2} \right) + n_{123} \left(\frac{n_2^k}{n_1^k + n_2^k + n_3^k} \right) \right] / \sum_{i=1}^n t_i$$

$$\hat{\lambda}_3 = \left[n_3 + n_{123} \left(\frac{n_3}{n_1^k + n_2^k + n_3^k} \right) \right] / \sum_{i=1}^n t_i$$

where $k = 1 + \frac{n_{12}}{n_1 + n_2}$.

With no masking these estimators reduce to the standard MLE's with the numerator of number of failures being divided by the total time on test observed. Note that with masking the numerator can be interpreted as an "estimate" of the number of failures caused by a particular component. Consider, for example, n_{12} . How many of n_{12} should we allocate as failures due to component 1? It is natural to consider an empirical allocation of

$$n_{12} \left(\frac{n_1}{n_1 + n_2} \right).$$

Similar comments apply to other terms in the numerators.

It is critical to note that these estimators are undefined when $n_1 + n_2 = 0$, i.e., no known causes of failures for components 1 or 2 are observed. We, therefore, restricted our study to consider only samples where $n_1 + n_2 > 0$. This condition also assured us of not allowing a sample with all the data masked. (Note this is analogous to censored data Monte Carlo studies where a sample with all censored observations is excluded.)

The entire simulation was programmed in FORTRAN and run on a VAX 11/750. The results are based on 100,000 repeated samples. This number of replicates was needed for very large masking proportions to assure an adequate number of samples not having every system being masked. This difficulty also lead us to simulate masking proportions only up to 95%. The results are graphed in Figures 1 and 2.

As expected, the bias and MSE get worse with increased masking. They improve with an increase in sample size from 10 to 100. For no masking (i.e., the masking proportion is 0) the three estimators are the standard MLE's with bias $1/(n-1)$. In Figure 1 we have graphed the baseline bias for each n for reference. For Figure 2 the reader could draw in the similar baseline MSE's by using the value at proportion 0. Note that the bias and MSE are fairly well behaved in spite of masking as large as 50% or 60% (even for $n=10$). For most industrial problems masking would be smaller than that.

Components 1 and 2 have bias and MSE that track each other as would be anticipated since they form a subsystem. Note that for rather large masking proportions, however, they become even more positively biased. To compensate for this, component 3 becomes strongly negatively biased. It seems for very

heavy masking that the numerator in $\hat{\lambda}_1$ (and $\hat{\lambda}_2$) may over assign masked failures as due to components 1 (and 2). (Recall earlier comments about the numerator is "estimating" the number of failures due to that component.) These aspects might motivate the search for modified estimators when the masking is very heavy. For $n=100$, however, the bias and MSE do rather well even up to 80% or 90% masking. From our industrial reliability analyses, we found the MLE's using L_R to be very reasonable.

5. CONCLUDING REMARKS

We have presented an approach which has actually been applied and found useful in a real world setting. Unfortunately, the data is not available due to proprietary rights. We had these problems arise in system life testing. We feel, however, the approach could also be useful in building better reliability data bases on components using field systems data (as well as life testing data on systems). Cf. the insightful paper by Doss, Freitag, and Proschan (1985) where they use complete (not masked) system lifelengths to estimate nonparametrically component reliabilities (some component lifetimes may be censored, while others could be complete).

In analyzing masked data, it is important to understand the mechanism causing the masking. If masking probabilities and conditions such as (2.2) and (2.3) are overlooked, estimators could turn out to be inconsistent. With careful attention to masking, valuable information can be incorporated properly for statistical analyses of key industrial devices.

When, for example, condition (2.2) is not true, how could a likelihood approach be developed? We are currently working on a modified method based on

the EM algorithm (see, e.g., Cox and Oakes (1984)) to accomplish that.

Finally, a likelihood development suggests building a Bayesian framework for analyzing masked data and the true cause of failure. We are also exploring this construction.

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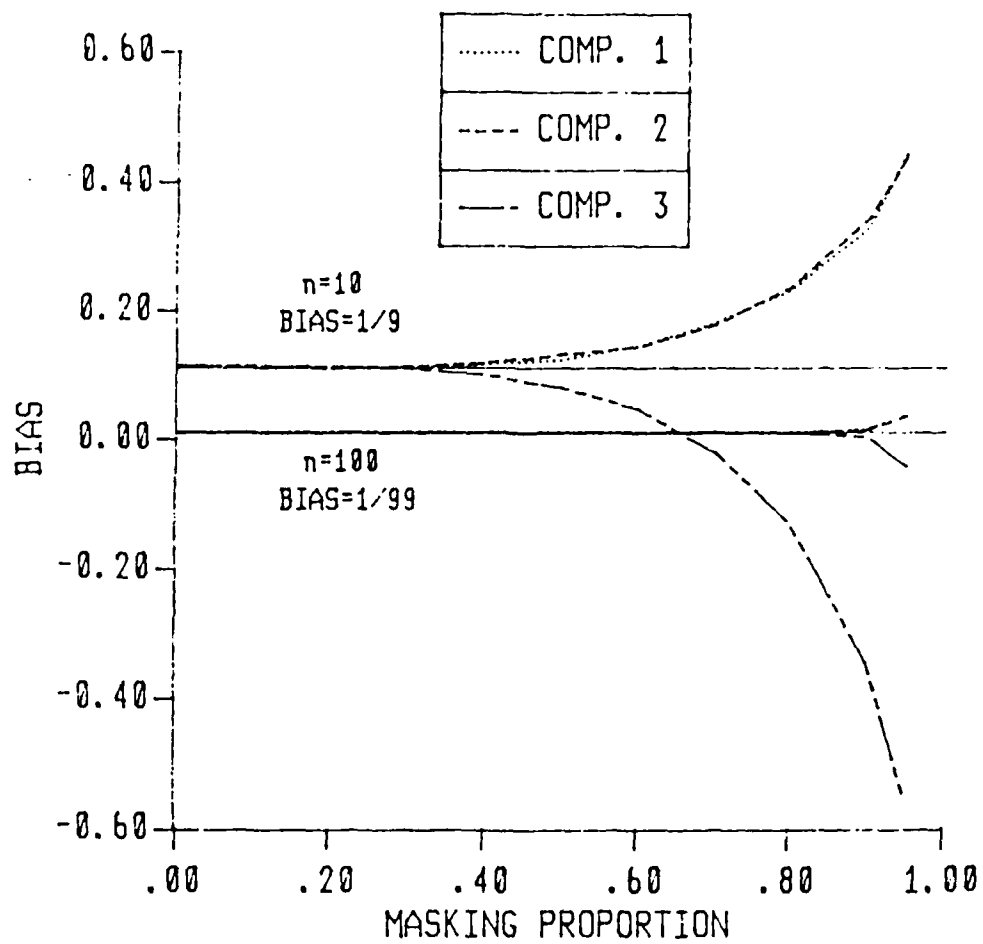
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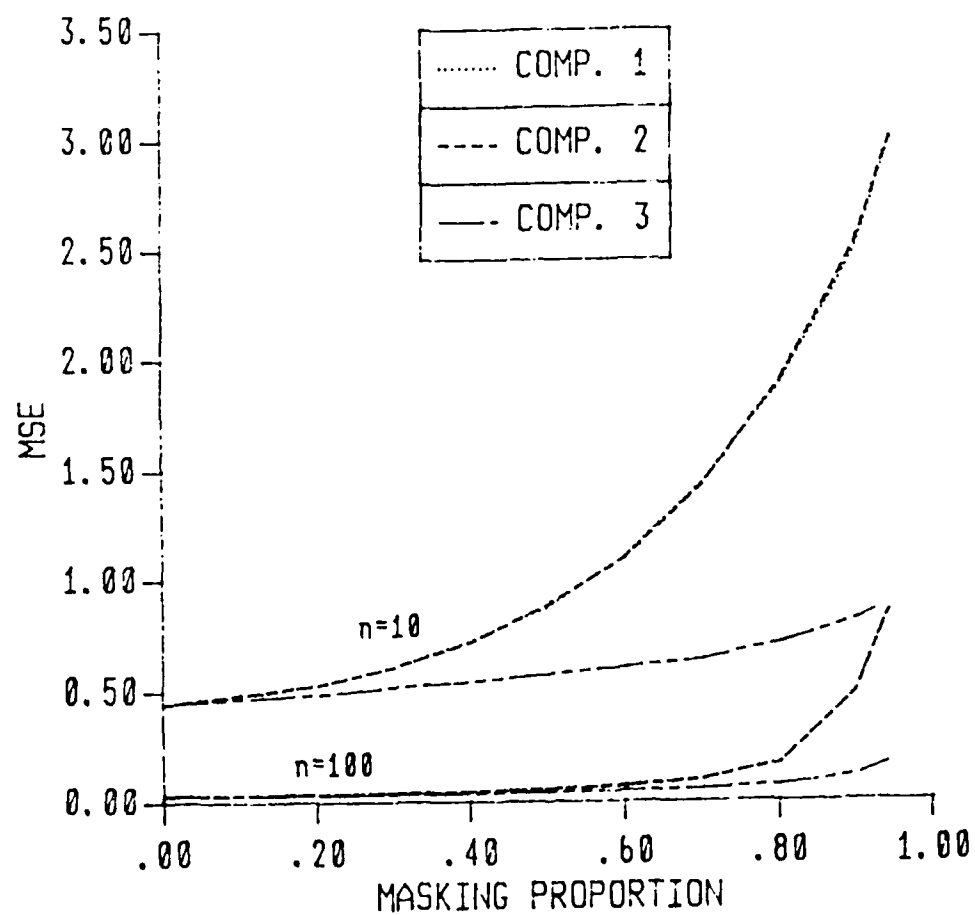
FIGURES

Captions

Figure 1: The effects of masking on bias

Figure 2: The effects of masking on mean square error (MSE)





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